Homework #1

due September 9

1 Reading

Please read Chapters 1–2 in your textbook.

2 Problems

Please do the following problem from the book:

2.2.7 (transitive closure) full natural language proof required

3 Pattern Matching

In this section, we will work with the following definitions of natural numbers and addition:

\[ m, n ::= z | s \ n \]

\[
\begin{align*}
\text{PLUSZERO} \\
& \quad z + n = n
\end{align*}
\]

\[
\begin{align*}
\text{PLUSSUCC} \\
& \quad n_1 + n_2 = n_3 \\
& \quad s n_1 + n_2 = s n_3
\end{align*}
\]

3.1 Part 1

If we wanted to prove \(0 + 3 = 3\) (that is, \(z + s s s z = s s s z\)), we would use the rule \text{PLUSZERO} with \(n = 3\) (or, \(n\) is \(s s s z\)).

If we wanted to prove \(2 + 2 = 4\) (that is, \(s s z + s s s s z = s s s s z\)), we would need to use rule \text{PLUSSUCC} where \(n_1 = 1\), \(n_2 = 2\) and \(n_3 = 3\) and we would need a proof of \(1 + 2 = 3\) as input.

Following these examples, explain what is needed to prove the following:

1. \(0 + 0 = 0\)
2. \(1 + 1 = 1\)
   - Don’t be put off by the statement being false; just give what rule would be used, what bindings to use for variables in the rule and what premises would be required.
3. \(2 + 2 = 2\)
4. \(1 + 2 = 3\)
5. Why is it impossible to do this exercise with \(0 + 1 = 2\)?
3.2 Part 2

If someone claimed to have proved $x + 0 = y$ using the rules above (where $x$ and $y$ are natural numbers unknown to us), they could have either:

1. Used PlusZero with $n = 0$ and $x = y = 0$.

2. Used PlusSucc with $n_1 = x'$, $n_2 = 0$ and $n_3 = y'$, $x = s\, x'$ and $y = s\, y'$ and a proof of $x' + 0 = y'$.

On the other hand, if they had claimed to prove $0 + x = y$, they could only have

1. Used PlusZero with $x = y = n$. In other words, $x$ and $y$ would have to be the same.

The PlusSucc rule cannot be used to prove an addition statement starting with zero.

Following these examples, indicate how someone claiming to have proved the statement could have proved it (what rule, with what variable bindings and what premises). Make sure that you only go one step back!

1. $1 + x = y$
2. $x + 1 = y$
3. $x + y = 1$.
4. $x + y = 0$
5. $x + x = y$.

4 Graduate Student Assignment

Find names of three “proof assistants” (not including SASyLF) used by computer scientists to mechanize proofs. Explain each in a few sentences. Give a URL of a published proof using each system. Do not use a sample/tutorial proof used to describe the software. It should be an independent proof done for the purposes of proving something in a published paper, not done for the purpose of demonstrating the proof system.

5 Submission

Turn this homework in on paper at the start of lecture.