Homework # 10
due November 11

1 Reading
Please read Chapter 16 in the textbook.

2 Problems
Please do the following problems

• Exercise 16.1.2
• Exercise 16.1.3
• Exercise 16.2.6

Check your answers against the solutions in the book, but do not turn in anything unless you have questions. Please do the following problems:

• Exercise 16.2.1
• Exercise 16.2.3

Turn in your answers to these problems.

3 Proofs
Proof the results about algorithmic subtyping: Proposition 16.1.5, Theorem 16.2.4, Theorem 16.2.5 in the given skeleton file.

These theorems rely on some important lemmas some of which are involved. In particular proving that algorithmic subtyping is transitive is tricky because we need to switch the order of the derivations in inductive calls. The textbook suggests using the “size” of the derivation as the induction argument. Doing so in SASyLF currently requires that the size be defined, and adds a large amount of arithmetic to the proof. There are two other ways to structure the proof:

1. One can prove two mutual inductive versions of the theorem, one doing induction on the first derivation, the other on the second, and they use mutual induction for the contravariant case of arrow types. Each will have to be accompanied by their own version of the theorem for record types. Unfortunately record subtyping does not treat the two sides uniformly; the supertype is treated in a different way than the subtype. This means that although the two versions of the theorem for types can share almost all the proof (copy and paste!), this doesn’t work for the record subtyping. In essence, one has to write three proofs.

2. One can prove the theorems using induction over the form of the “middle” type (not a derivation). Since the middle type is always in the middle, even when we have the contravariant case, we need only the one theorem (and its mutually induction record version).

4 Graduate Students
Find a published paper in the last ten years that talks about algorithmic typing or subtyping. Give the citation (as a link to the ACM digital library) and explain the situation. Explain the issues involved in showing soundness and completeness.