Homework # 6  
due October 14

1 Reading

Please read Chapter 9 in the textbook.

2 Discussion

In class, we discussed several dubious type rules. In particular, we discussed something similar the following system:

\[
\begin{align*}
\text{T-VarA} & : x : T \\
\text{T-AbsA} & : t : T \\
\lambda x.t : T \to T \\
\text{T-AppA} & : t_1 : T \to T' \\
t_2 : T \\
(t_1 t_2) : T'
\end{align*}
\]

... additional rules for booleans and “if” ...

Suppose we try to prove progress and preservation (for closed terms) with this “type system.” Can we do it? Explain! Give concrete examples.

If we changed the rules to

\[
\begin{align*}
\text{T-VarB} & : x : \text{Bool} \\
\text{T-AbsB} & : t : T \\
\lambda x.t : \text{Bool} \to T \\
\text{T-AppB} & : t_1 : T \to T' \\
t_2 : T \\
(t_1 t_2) : T'
\end{align*}
\]

... additional rules for booleans and “if” ...

Can we prove progress and preservation? Discuss!

3 Proofs

Read Exercise 9.2.3 and prove that typed self-application is impossible in the simply-typed lambda calculus, that is, if $\Gamma \vdash t t : T$ then we have a contradiction. Also do the proof of preservation (Theorem 9.3.9). You will need to use the substitution lemma (Lemma 9.3.8). You can use the solution in the back of the book, but should realize that the $\text{E-AppAbs}$ case skips a step.

4 Extra

In the skeleton file, some of the helper lemmas are marked as “EXTRA.” Only if you complete the rest of the homework, you are invited to do these proofs. A better grade on this part will replace a poorer grade for the SASyLF part of a previous homework.

5 Graduate Students

In the substitution lemma, we need to use “weakening” and “exchange” lemmas/rules. What flavor of type systems does not have weakening? Explain! When is exchange for some type systems problematic? Explain!